DIFFUSER OPTIMUM IMBIBITION

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Abstract

Sugar is extracted from cane in a diffuser, in which crushed cane is passed along a conveyor and water is recycled through a number of different sections to wash out the sugar. The study group was asked to consider the amount of water to be included in the diffuser system to optimise the extraction of sugar. A simple model was derived that computes the exchange between the cane and juice and computes the resulting concentrations. The model was based on one derived at the South African Study Group in 2012 [2]. Simulations were conducted in which water was added at various stages along the process. The amount of water added at different stages was varied to consider a range of different strategies. It was found that it was possible to optimise the removal of sugar while minimising the amount of liquid to be burned off post-process, but the relative costs of this require further analysis.

1 Introduction

Sugar cane typically consists of 70% water, 15% fibre and 15% dissolved solids (mainly sugar). Optimum sucrose extraction is essential for the sugar industry. The sugar-milling process begins with the shredding of sugar cane which reduces the cane to short fibres. The shredded sugar-bearing cane is then passed through a diffuser which is a device consisting of 12-14 stages. Extraction of sugar-containing juice

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from the cane is facilitated by the addition of water by way of sprays positioned at intervals on the diffuser. Additional water, known as imbibition, is introduced in the final stage of the diffuser in order to maximise the extraction process. Water that was added has to be evaporated before sugar is crystallised. This evaporation process requires energy by way of burning fuel in a boiler. Much of this fuel comes from the spent shredded cane. A drawback of imbibition is that the evaporation of this additional water increases the fuel consumption and consequently the production cost, as coal needs to be bought when all the fibre has been used up. Thus, in order to reduce cost, a possible solution would be to reduce imbibition. However, would a reduction of imbibition result in lower extraction efficiencies? This is the question we explored in our work.

A detailed model of the exchange between the cane and the juice was considered in the South African study group in 2012 [2]. This model was able to accurately predict the concentration in the diffuser when compared with the measured data. Subsequent projects considered the water levels in the diffuser [4, 5]. These projects have led to a good understanding of the flows in the diffuser, and so it is possible to develop some optimal strategies. In all of this work the assumption was made that the flow of the juice through the crushed cane can be treated as a flow in porous media, [1, 3] and this assumption has been proven to be effective.

In this report we have derived a simple model that computes the exchange and then computes the concentration in the diffuser bed and the juice given water added at various stages along the process. This is more than was asked in the initial project, in which only a consideration of imbibition at the last stage of the process was requested, but it seemed worthwhile to consider further, similar possibilities. Once this model was set up, we were able to test different strategies for adding water at different stages to see if it was possible to optimise the removal of sugar while minimising the amount of liquid to be burned off post-process.

2 A model

We obtained a system of equations which allowed us to investigate this from a perspective of flow rates. We formulated a simple discrete model, utilising a 12-stage diffuser. We tracked the concentration of sugar in the cane and in the juice as it made its way through the different stages. Here, we focused on the vertical velocity of the liquid in the diffuser and were able to establish that a lower vertical velocity (and thus a lower vertical flow rate) produces a higher concentration of sugar in the juice. However, given a particular set-up of the diffuser and value of permeability, the amount of water/juice that can flow through the bed is determined by the properties of the cane, and so to compare situations we may assume a constant flow rate and rate of exchange of sucrose between the juice and the cane.

2.1 Standard process

In this section we will obtain a baseline model in which the water is re-cycled without any addition of fresh water. This is a simplified version of the model of Breward et. al. [2]. We will then use this to consider modifications that might improve the efficiency of the process. If we define S_n, C_n, $n = 1, 2, \ldots, M$ where M is the number of cells, to be the concentration of the juice and sugar at the top of the cane bed in each cell in the cane respectively, then we can say that

$$S_{n-1} = \alpha_n S_n + \beta\Delta t(C_n - \alpha_n S_n) \quad (1)$$

$$C_{n+1} = C_n - \beta\Delta t(C_n - \alpha_n S_n) \quad (2)$$

for $n = 1, 2, \ldots, M$, where α_n is the dilution of the water (if we were to siphon off some of the juice and replace it with a small amount of fresh water) from the previous cell (downstream), β is the exchange constant between the juice and cane, and $\Delta t = h/k$ is the approximate time taken for the juice to percolate from the top to the bottom, where h is the depth of the diffuser bed and k is the intrinsic permeability of the bed. In the "standard" model, in which the process runs without any intervention, $\alpha_n = 1, n = 1, 2, \ldots, M$. This is a simplified version of the model proposed and verified in the 2012 study group [2]. In this simpler model, each cell is assumed to be a parallelogram with no exchange with the next cell. Thus the liquid flows in the top and departs into the appropriate tray with no flow into the next cell. The model in Breward et al [2] gave an exact solution and the model proposed in this report also has an exact solution if α is kept constant throughout. However, since we wish to investigate various strategies with different values of α for different cells, the problem no longer has an exact solution, and so we go straight to the numerical solution.

Writing the model for $M = 3$ (say) we obtain a matrix system of the form

$$\begin{pmatrix} 1 & 0 & 0 & -\alpha_1\theta & 0 & 0 \\ 1-\theta & 1 & 0 & 0 & -\alpha_2\theta & 0 \\ 0 & 1-\theta & 1 & 0 & 0 & -\alpha_3\theta \\ -\theta & 0 & 0 & 1 & -\alpha_2(1-\theta) & 0 \\ 0 & -\theta & 0 & 0 & 1 & -\alpha_3(1-\theta) \\ 0 & 0 & -\theta & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} C_2 \\ C_3 \\ C_4 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \begin{pmatrix} (1-\theta)C_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ (1-\theta)S_4 \end{pmatrix} \quad (3)$$

where $\theta = \beta\Delta t$, so that we can see the pattern of the matrix. We note that C_0 is the initial concentration of sugar in the cane, and $S_M = 0$ is the concentration of the sugar in the water. This matrix is not quite tridiagonal, but could be solved in the same way. However, it is a simple matter to solve this matrix equation using Octave to provide the concentration in the different cells along the diffuser.

Figure 1 shows the results for a "standard" case. It is clear that slopes of the two lines for the concentration are constant and parallel. In this case, about 85% of the sugar is removed from the megasse. We take this as a control case so that

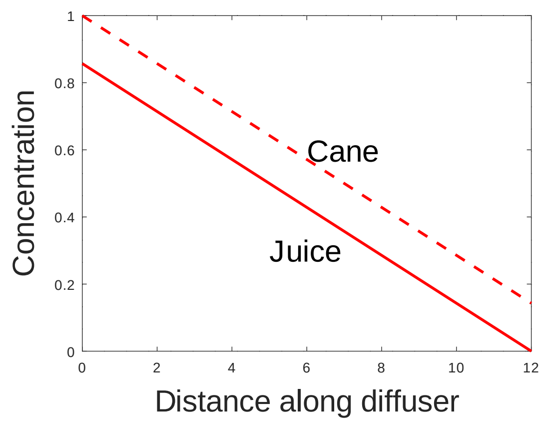

Figure 1: *Simulation of concentration in the juice (solid line) and the cane bed (dashed line) for the case where the exchange coefficient* $\beta = 0.5$ *is such that around* 85.9% *of the sugar is extracted. In this example the liquid is recycled 12 times.*

we can see the affect of minor dilution of the juice in the early cells. It is clear that there is a linear relationship between the juice concentration and the concentration in the megasse. As a simple numerical experiment, we also included an extra recycle into the process, so there were 13 instead of 12. As a result only a very small extra amount of cane was removed (less than 1%), suggesting that 12 cells is sufficient.

2.2 Modified process

In this case, we allow the value of dilution of $\alpha_k, k = 1, 2, \ldots, M$ to vary. The idea is that some of the water/juice being recycled is siphoned off between cells and replaced with fresh water to increase the difference in concentration between the juice and the cane. The introduction of this water will increase the amount of sugar collected, but will also increase the amount of water that needs to be "boiled" off after the process is complete, and so there is an optimisation problem in determining the cost of extra boiling compared with extra amount of sugar. It is therefore of interest to minimise the addition of water.

In the first example, shown in Figure 2 the first three cells in the diffuser have 5% of the juice removed and replaced by fresh water, thus increasing the exchange in this first cell. The result is that about 88.8% of the sugar is extracted by the end of the process, an increase of 3% on the control with 15% extra liquid.

In the second example, shown in Figure 3 the juice is diluted by 10% in just the first cell (as the cane enters). The result is that around 87.3%, an extra 1.6% of the sugar is extracted beyond the control with a 10% increase in liquid.

Finally, a 10% dilution is applied on each of the first two cells. The result is given in Figure 4, and shows that 89.1% of the sugar is removed. This dilution results in approximately 20% extra liquid.

Table 1 gives a summary of the effectiveness of the different strategies in terms of

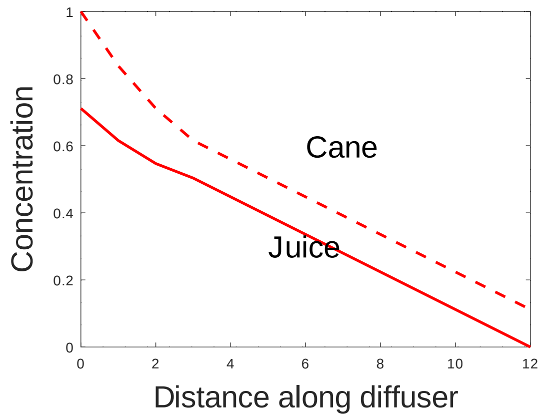

Figure 2: *Simulation of concentration in the juice (solid line) and the cane bed (dashed line) for the case where the exchange coefficient* $\beta = 0.5$ *as in Figure 1, but in this example the juice is diluted by 5% in each of the first three cells (as the cane enters). The result is that around* 88.8%*, an extra 3% of the sugar is extracted.*

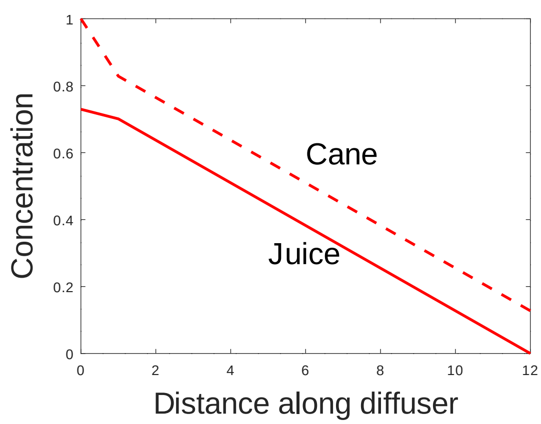

Figure 3: *Simulation of concentration in the juice (solid line) and the cane bed (dashed line) for the case where the exchange coefficient* $\beta = 0.5$ *as in Figure1, but in this example the juice is diluted by 10% in just the first cell (as the cane enters). The result is that around* 87.3%*, an extra 1.6% of the sugar is extracted beyond the control.*

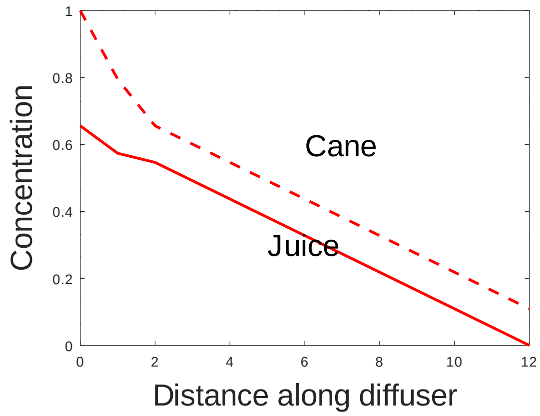

Figure 4: *Simulation of concentration in the juice (solid line) and the cane bed (dashed line) for the case where the exchange coefficient* $\beta = 0.5$ *as in Figure 1, but in this example the juice is diluted by 10% in each of the first two cells. The result is that around* 89.1%*, an extra 3% of the sugar is extracted beyond the control.*

Table 1: Table of the effectiveness of different dilution strategies for 12 cells, with $\beta = 0.5$.

Dilution strategy	% sugar removal	Extra liquid
Control	85.7%	0
10% cell 1	87.3%	10%
15% cell 1	87.9%	15%
5% cells 1-3	88.8%	15%
4% cells 1-4	89.3%	16%
10% cell 1-2	89.1%	20%

the percentage of sugar removed and the extra liquid. Adding more water increases the removal, but it is not straightforward. For example, adding 4% water in each of the first 4 cells, for a total of 16% extra water, produces a better result than simply taking 15% from the first.

3 Comments

The task was to determine if the amount of sugar removed could be increased by adding extra fresh water to the process, and whether decreasing the amount of imbibition would lead to a decrease in extraction. However, the flow of water through the system is determined purely by the properties of the megasse, so only a fixed amount of water can flow through. Therefore, in order to add more fresh water, some of the existing cycled juice would need to be siphoned off and replaced as the process runs through. The siphoning equations that determine the sugar extraction and juice concentration were programmed, and then several strategies of adding water (after siphoning off the equivalent amount of juice) were tested. It is clear that this process is effective in increasing the sugar removal, but at the expense of the extra energy required to boil off the excess moisture. The question is then whether the cost of this extra energy is greater or less than the profit from extra sugar production. This may depend on a number of factors including current costs and prices. Conversely, reducing the amount of fresh water added will clearly decrease the amount of sugar extracted. Therefore it again comes down the relative profits of the extra sugar compared to the losses due to evaporating the excess water. It seems adding slightly dilute water as the megasse enters the diffuser is more effective than “washing” as it exits the diffuser, although it is clear that some physical modification to the diffuser set-up would be necessary, and the increase in sugar production may be small.

The work has shown that it is possible to increase the sugar produced, and a method is described that would allow this calculation to be made and an optimal outcome achieved.

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